Solve the following system:

$$\begin{cases} \frac{3}{x}+\frac{4}{y}=4 \\ \frac{5}{x}+\frac{2}{y}=16 \end{cases}$$

This looks crazy! We don't want to solve with variables in the denominator. What can we do?

Step 1 – Make new variables equal to the fraction variables

Let's use m and n. They don't appear anywhere in our original problem, but that's what we want. Here is what m and n will equal…

$$m=\frac{1}{x} \quad \text{and} \quad n=\frac{1}{y}$$

Be sure to write this step on your homework!

Step 2 – Substitute the new variables into the system of equations

Our original system could be rewritten like this:

$$\begin{cases} \frac{3}{x}+\frac{4}{y}=4 \\ \frac{5}{x}+\frac{2}{y}=16 \end{cases} = \begin{cases} 3(\frac{1}{x})+4(\frac{1}{y})=4 \\ 5(\frac{1}{x})+2(\frac{1}{y})=16 \end{cases}$$

You may write this step on your homework, but it is **not required**

Let's substitute our new variables, m and n, in for $\frac{1}{x}$ and $\frac{1}{y}$.

$$\begin{cases} 3(\frac{1}{x})+4(\frac{1}{y})=4 \\ 5(\frac{1}{x})+2(\frac{1}{y})=16 \end{cases} = \begin{cases} 3m+4n=4 \\ 5m+2n=16 \end{cases}$$

You will write this "new" system on your assignment

Step 3 – Solve using the substitution or linear combination method (you choose).

For this example, we'll use linear combination and make the variable n drop out.

$$\begin{cases} 3m + 4n = 4 \\ -2(5m + 2n = 16) \end{cases}$$

$$\downarrow$$

$$\begin{cases} 3m + 4n = 4 \\ -10m - 4n = -32 \end{cases}$$

$$-7m = -28$$

$$m = 4$$

$$3(4) + 4n = 4$$

$$12 + 4n = 4$$

$$4n = -8$$

$$n = -2$$

Step 4 – Solve for the original fraction variables

We found $m = 4$ and $n = -2$, but we really need to know about x and y. Remember

$$m = \frac{1}{x} \quad \text{and} \quad n = \frac{1}{y}$$

That means

$$4 = \frac{1}{x} \quad \text{and} \quad -2 = \frac{1}{y}$$

$$4x = 1 \quad \text{and} \quad -2y = 1$$

$$x = \frac{1}{4} \quad \text{and} \quad y = -\frac{1}{2}$$

Step 5 – Write solution as ordered pair.

$$\left(\frac{1}{4}, -\frac{1}{2}\right)$$